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Some Misunderstandings Related to WTC Collapse Analysis

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ABSTRACT
This article elaborates on variables associated with the collapse of the North Tower of the World Trade Center. The previously published quantifications of inertia, column capacity, and the assumptions related to the beginning of downward motion, are examined and corrected. The reasons for false conclusions reached in several previous analyses are presented.

Key words: Large Deflections, Plasticity, Collapse

1. INTRODUCTION

This presentation is not so much about how the WTC towers failed, but about *how they could not* fail. The objective is to eliminate erroneous concepts supported by false assumptions and by the use of incorrect values for velocity, mass, and column resistance. The only complete hypothesis of the global collapse mechanism of the Towers is a successive flattening of stories associated with compressive column failure and referred to as a Progressive Column Failure mode or PCF in brief. (In the past this mode was often referred to as *pancaking*, but this term is not used here to avoid ambiguities). It is explained here why PCF could not be the mode of the ultimate destruction. The previously published material is quoted and the new points are brought up. Appendix C can be of interest to those who want a broader description of facts associated with the collapse. The available information relating to the kinetics of the collapse is summarized first.

2. THE FIRST PHASE OF DOWNWARD MOTION

A good comparison between various collapse models and reality makes it necessary to have some observations of the towers during collapse. To our knowledge, the most accurate and reliable data available are provided by video footage taken by Etienne Sauret [7], and used in the documentary film *WTC: The first 24 hours*. This footage clearly shows the top of WTC 1, including the roof line, for about the first 3.2 seconds of the collapse. Each pixel represents 0.27 m of the tower, and the frame rate is 30 per second, allowing for fairly accurate measurements of the motion. It is

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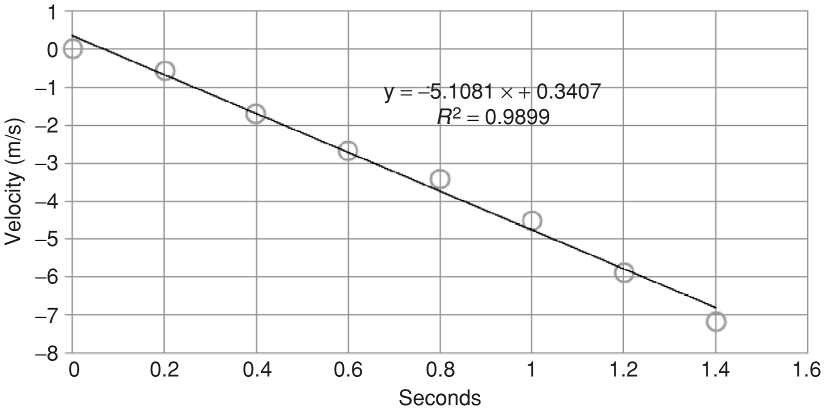

Figure 1. Measured initial velocity of the North Tower fall

unfortunate that the roof line is visible for only 3.2 seconds before disappearing into a dense cloud of debris, but these few seconds are, in fact, quite informative.

Now, let us have a look at the numerical values. The velocity of the falling roof of the North Tower was measured in [7] and the initial phase over the first 1.4 seconds is shown in Figure 1.

The averaged acceleration during the early phase of the fall (shown as the slope of the velocity curve in Figure 1) was approximately 5.11 m/s^2. The resulting velocity after 1.2 seconds of the fall, which is the approximate time for a fall of one story (3.7 m), is 6.13 m/s (13.71 mi/h).

For comparison, we note that a free drop (acceleration of 9.81 m/s^2) of an object from a height of 3.7 m takes 0.869 s and the peak velocity reached then is 8.52 m/s. The total height of the tower was 417 m and the time needed for a free drop from that height was 9.22 s, while the end velocity of the dropping object would reach 90.45 m/s. If the drop is counted to only the top of mezzanine, at 21.33 m above ground (a notional top of the rubble heap), then the time is only 8.98 s.

3. COLUMN RESISTANCE IN LARGE-DEFLECTION COMPRESSION

The upper limit of compressive strength of steel columns is usually taken as $P_y = AF_y$, where F_y is the yield strength and A is the section area. This value is reduced to the buckling load P_{cr}, according to column slenderness [14]. However, if the columns are stout, as in the case here, the difference between P_y and P_{cr} is usually quite small. The peak resistance is encountered at the outset, when a column is straight.

One of the methods used to assess the resistance offered by a slender column past the elastic range is to treat it as a three-hinge mechanism as done by Bažant [1, 2]. The resisting force decreases with deflection until the two arms of the mechanism become horizontal. At that point there is a minimum resisting force, which is only a small fraction of its initial P_{cr}. One can expect reasonable results from such a mechanism in a slender column, but only in the early stages of deflection. When interactions between the walls of a column develop, secondary resistance arises.

Because the columns in the WTC towers were stout by any criterion, one can expect that, in the large deformation range, the columns would exhibit a minimum resistance being a significant fraction of P_{cr}. Consequently, Szuladzński [3] employed the same approach, as that of Bazant described above, except that he placed a higher estimate on the resisting capability of columns. With this, he concluded that arrest of the downward motion would take place quite promptly, just outside the zone affected by the aircraft impact.

A further insight into the level of resistance by stout columns was offered by Szuladziński [5], who conducted a FEA simulation of a large deflection squashing of a rectangular, hollow section column (RHS). If P_{av} designates the average resistance, the result can be written as

$$P_{av} = \eta\, P_{\mathrm{cr}} \tag{1}$$

where P_{cr} is the buckling load and η is the retention factor. For the examined RHS it was found that $\eta = 0.4$, which was more than assumed in [3]. Prior to that, a square hollow section (SHS), was used in a simulation of a large-deflection squashing, Ref.[10]. The retention factor there was not less than $\eta = 0.665$, with this figure related to the use of the minimum, not the enhanced value of F_{y}. (In this and the previous example the material was treated as strain-rate insensitive, but there was some increase in the apparent strength due to dynamic application of the compressive load.)

Another study, conducted by Korol [4a], involved closed- and open-section beams. Using semi-experimental formulas by Wierzbicki and Abramowicz [4b] it showed the resistance levels in the $\eta = 0.4$ range or somewhat below. It is anticipated that thinner open-section beams may have $\eta < 0.4$, and probably closer to $\eta = 0.3$. A detailed calculation of the nominal column strength of the assembly and the expected resistance is provided in Appendix A.

The above estimates are conservative, as they do not take one important factor into account.

When the vertical deflection exceeds some 2/3 of the length, the walls of a column are folding and leaning on one another and possibly on the floor. As the process continues and deflection progresses, it leads to quite large resisting forces being developed. The simulations done so far were not usually carried out to large enough deflections to take full advantage of this effect.

Buildings are designed against gravity and lateral loads. Column-and-slab structures are not very efficient in resisting the second loading type, as the lateral forces cause differential bending of columns. For this reason, much more material is used in designing columns than would be needed for resisting gravity alone. When progressive collapse is considered, only gravity is involved, which means that there is a large safety factor against purely gravitational loads. This is the reason why a complete failure according to the PCF mode is generally not a practical proposition. This is not to say that the mode could not occur during a small part of the collapse, followed by an arrest.

The calculation of movement and arrest mentioned previously (Ref.[3]) referred to the core alone and was based on a certain load distribution between the core and the outer shell. That work was therefore limited, because the original PCF concept was for the building as a whole. Additionally, not all relevant details were established at the time. For this reason a global approach will be presented below.

4. QUANTIFICATION OF PROGRESSIVE COLLAPSE IN PCF MODE

The National Institute of Standards and Technology (NIST) determined that the North Tower failure initiation occurred at the 98th story [9], putting the upper section size at 12 stories. The actual mass of this 12 story section can be found in the NIST report (NCSTAR 1-6D, p. 176, Table 4–7), which states the actual total load on the columns between floors 98 and 99 to be 73,143 kips, i.e. 325.4×10^6 N or 33.18×10^6 kg. This is corroborated by the independent analysis shown in [8].

The motion began when the strength of the columns on the critical floor fell just below the weight those columns were supporting, therefore $P_{av} = W = 325.4 \times 10^6$ N. This was an average strength in the assembly of columns involved. Many of those columns must have been damaged, bent out of shape and distorted in various ways. Those that were intensely heated would increase their deviation from straightness. Probably only a minority of columns retained the straight-line axis. It was not much of buckling in the traditional sense that followed. It is an open question as to what the characteristics of such columns could be. However, there is a clue in Fig.1, which indicates near-constant acceleration, typical of near-constant resistance. As shown in [3] as well as below, the above P_{av} is itself only a small fraction of the undamaged buckling strength, so it seems reasonable to use it, in the first approximation, as a typical, average resistance along the downward path. (For this reason P_{av} was used here rather than P_{cr}.) The energy absorbed by columns over the first story travel is then

$$\Pi = P_{av}\, h = 325.4 \times 10^6 \text{ N} \times 3.7 \text{ m} = 1{,}204 \times 10^6 \text{ N-m}.$$

The potential energy for a one-story drop is:

$$U = Wh = 325.4 \times 10^6 \text{ N} \times 3.7 \text{ m} = 1{,}204 \times 10^6 \text{ N-m}.$$

Both quantities are identical, therefore the arrest takes place over the critical story. One can also choose to be more conservative and degrade the average resistance by another 25%. Thus, the energy absorbed over the first story travel will be

$$\Pi = 0.75\, P_{av}\, h = 0.75 \times 325.4 \times 10^6 \text{ N} \times 3.7 \text{ m} = 903 \times 10^6 \text{ N-m}.$$

This time $U > \Pi$, therefore stopping is not complete over this story and further motion must be considered. Noting that the gross kinetic energy available to overcome the resistance of the next story below is

$$E_{\mathrm{k0}} = 1{,}204 - 903 = 301 \text{ MN-m}$$

Before further motion continues, there is an energy loss incurred due to the accretion of the slab. When this is treated as a fully plastic collision, that loss, according to [5] is

$$\Delta E_k = \frac{1}{2}\left(\frac{M_s}{M_s + M_b}\right) M_b v_0^2 = \left(\frac{M_s}{M_s + M_b}\right) E_{k0} \tag{2}$$

where $M_{\mathrm{b}} = 33 \times 10^6$ kg is the mass of the descending part of the building and $M_{\mathrm{s}} = 2.74 \times 10^6$ kg is the mass of the accreted slab and its tributaries. After substituting, one finds $\Delta E_{\mathrm{k}} = 23$ MN-m. Consequently, at the outset of travelling down by a further story, the following status develops. The moving part now has $M = (33 + 2.74) \times 10^6$ kg $= 35.74 \times 10^6$ kg and its potential energy relative to the next floor is $U = Mgh = 1{,}297.3 \times 10^6$ N-m. The energy available to overcome the column resistance is now

$$E_{\mathrm{k0}} - \Delta E_{\mathrm{k}} + U = (301 - 23.0 + 1{,}297.3) \times 10^6 = 1{,}575.3 \times 10^6 \text{ N-m}.$$

The energy absorption capacity of undamaged columns is estimated, in Appendix A, as 2,720 MN-m. The NIST report predicts up to 20% of the core columns to be severed or severely damaged in the North Tower and it is known from observation that approximately 15% of the perimeter columns were severed or severely damaged. To be conservative, the remaining capacity of the story below the critical one can also be assumed degraded. Even if only three-fourths of the undamaged capacity is preserved, we are left with 2,040 MN-m. This is more than the total energy available, 1,575.3 MN-m, which indicates the arrest of motion during travel along this story.

5. FALSE CONCEPTS STILL CIRCULATING

As mentioned before, one can expect reasonable results from a hinged post-buckling mechanism if a column is slender and then only in the early stages of deflection. With an advancing deformation pattern interactions between the walls of a column develop, with this being strongly marked for stout columns. Yet, the results from said mechanism were taken by Bazant [2], somewhat naively, to hold over the entire squashing motion. That gave the column resistance as asymptotically decreasing with deflection, which is an unrealistic proposition.

The main paper proposing the PCF mode [2] had several discussers involved. The closure of the discussion [15] pointed out a few flaws that crept into the competing analysis [3], although these flaws had no adverse effects on the results. The defense of the analysis presented in [2], on the other hand, was essentially relying on ignoring the arguments to the contrary. The gross underestimate of the energy absorption capacity of the squashed column that resulted is named here as the fatal mistake No.1.

From the above assumption Bazant concluded that after the initiation of failure of the critical story, the columns in that squashed story offered only negligible resistance. That led to the assumption that the upper part of the building, above that story, would be in a free fall until a complete flattening of that story. At the end of the critical story squash the free-drop velocity is over 8.5 m/s, resulting in a destructive impact. This in turn culminated in [2] as a conclusion of a quick collapse of the entire edifice. We call this a "vanishing story assumption" and refer to it as the fatal mistake No.2. (This free-fall assumption was not openly stated, but there are numerous hints in [1, 2, 6] and [12] implying that it was used in computational procedure.)

Thus, in 2008, the PCF mode was demonstrated to be an inadequate explanation of the building collapse. Yet, some insist on ignoring the numbers. A recent publication [6] implies that [2] was entirely correct and says: *The collapse of the World Trade Center towers was initiated by the impact of the upper part falling onto the underlying intact story.* The use of Eq.1 in [6] amounts to assuming a free drop, by one story.

This free drop is inconsistent with a generally agreed scenario, according to which during the initiation of the collapse the strength of the critical story had to be gradually weakened by fire to the point where its strength was reduced to being unable to carry the weight above. From then on the resistance of the buckled columns was gradually decreasing, but would still be significant with respect to the load above them.

When a calculation consistent with a PCF is made, as demonstrated above, the impact velocity drops to a small value indicating an arrest or nearly so, so there is no need to explain an abrupt impact as attempted in Note [6]. A small change in the velocity of the downward-moving mass on such impact is a trivial point, as the reader may find above comparing ΔE_k resulting from such an impact with the initial potential energy U. Thus, there is no need to write a paper to explain why that contact of the upper and the lower part of the building was not noticeable on the video records. This observation removes the whole premise of Note [6].

To keep in contact with reality, one should note that Fig.1 shows the measured velocity developed at the end of the fall over the critical story to be much larger (6.13 m/s) than the speed implied by the PCF mode. This is simply another indication that PCF is not a proper explanation of the process.

What the authors of [2] and [6] clearly failed to appreciate was the difference between the softened structure and a vanished structure. One should also note that if there were any grounds to assume the concept of a "vanishing story" as true, the arrest of motion would still be concluded, although after a longer travel.

Another problem with [2] is the lack of transparency in showing the reader how the results were obtained. In a way, the paper is a display of mathematical brilliance by formulating several arrays of differential equations. There are a few clues given as to how those equations *could* be solved, but nothing is said on how the solution *was* actually obtained. The author says that for a typical tower configuration the fall time is 10.8 s (*vs*. 9.21 s for a free fall). For all of the wide range of parameters (including a structure stronger and lighter than the real one) the fall time was reported to be less than 18.4 s. This sounds strange, when considering the results of a more recent work [5], where the calculated time was 15.3 s while assuming *no* resistance from columns. The conclusion from all of this is that the results of [2] were, at best, grossly in error because of the flaws mentioned earlier, or, at worst, processed to match the lower bound of the observed fall time.

6. CRITERIA FOR VERTICAL BUILDING COLLAPSE, BASED ON COLUMN FAILURE

Two criteria must be met for any transient, dynamic event that changes the state of the system to take place. The first is the presence of a big enough force to make that change possible. The second states that there must be enough energy supplied to enable the change of state. This collapse is no exception. The first criterion says the gravity force acting on the top part of the building must be slightly larger than the strength of the story below that top part. (Or, that a story must have been weakened to a degree meeting that condition.) The second one comes from the energy balance and determines how long that collapse will last. It says that collapse will continue as long as the kinetic and potential energy of the downward-moving portion of the building exceeds the energy-absorption capacity of the columns of the story below.

The texts of the criticized papers mention both criteria, although in a confused manner. In [2], a statement on p.311 tells the reader that, in fact, the energy criterion renders the strength criterion irrelevant. The conclusions echo that by saying: *What matters is energy, not the strength, nor stiffness.* Confronted with the results of discussion [3], showing a prompt arrest, Bazant took a small step backward in Closure [15] by saying that the strength criterion was valid in statics, but not in a dynamic environment, where inertia forces are involved. Thereby the author missed the fundamental point, namely that *the initiation of motion took place in a static environment and therefore meeting the strength criterion was indispensable.* (Dynamics enters later, when significant accelerations or inertia forces are developed.) Conclusion: The results were obtained while ignoring the strength criterion and are therefore invalid on this count alone. (This was a visible confusion between statics and dynamics and is regarded here as the fatal mistake No.3.)

7. OTHER PROBLEMS WITH THE HYPOTHESIS OF PCF MODE

It is difficult to understand why the inflated mass of 54.18×10^6 kg was used in [2] and [6], and an even larger value of 58×10^6 kg applied in [1], when the per-floor mass was correctly described in the addendum to [1], and also, when one of the authors was common to all three papers. In that addendum the mass of 44% of the whole tower was calculated as 141×10^6 kg using a modal frequency analysis. This equates to 320.45×10^6 kg for the whole tower. When this mass is divided by 117 floor levels (for 110 above-ground stories, 6 sub-levels, and the roof) it gives a single floor mass of 2.74×10^6 kg. Multiplying this per floor mass by the 12 stories in the upper part of the North Tower gives it a mass of approximately 33×10^6 kg.

Let us designate the kinetic energy at first impact by K and the energy absorption capacity of the columns by $\prod$. When the velocity is corrected from 8.52 to 6.13 m/s, the mass from 58×10^6 kg to 33×10^6 kg, and $\prod$ from 500 MN-m to a minimum of 1,686 MN-m as shown in Appendix A , the value of $K/\prod_{\min}$, given implicitly as 4.2 in Ref. [1] for a one story fall, is found to be significantly lower, namely 0.368. A gross over-estimate of the relative magnitude of the kinetic energy available vs. column energy absorption capacity was the reason for concluding in [2] that the motion of the upper part was unstoppable.

As one can readily see, this mode of damage is a distinct process, whereby each floor becomes crushed, one after the other. An attempt has been made to smear the process out into a continuum event and then use differential equations to obtain a solution [2]. Yet, justification of such an approach must be performed, by showing that little is lost in translation. As no such justification exists, the whole approach in [2] seems questionable.

8. SUMMARY

A number of simple, transparent calculations of the North Tower collapse were presented in [5] and the conclusion was that assuming even a modest resistance of columns during their destruction would cause an unacceptably long collapse time. It is only when perfectly frangible columns were adopted that the fall time was as low as 15.3 s. This removes the PCF mode, as defined here, as a viable hypothesis of collapse.

Yet, the PCF achieved significant popularity, as based on [1] and [2], while the next work [12] did not contribute anything new to the core of the subject. These papers, purporting to explain the collapse, suffered from three fatal errors, as detailed above. Also, the whole methodology was not justified. Some incredibly short fall times were quoted by the authors, while all solutions were of a black-box type. The presentations in these papers are not a valid description of what happened. The reasons for a smooth motion history and promptness of collapse of the North Tower remain yet to be determined.

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APPENDIX A. ESTIMATED STRENGTH OF COLUMNS

The columns of the outer shell of the building were square hollow sections (SHS) of 356 mm (14'') side length. At the 98th and the 97th floors the wall thickness was 6.9 mm (0.270'') and the yield strength was 65 ksi (448.3 MPa) according to the NIST report. There were 240 such columns and their total cross-sectional area was 2.296 m^2. The yield load was thus

$$P_{y1} = 448.3 \text{ MPa} \times 2.296 \text{ m}^2 = 1{,}029 \text{ MN}$$

The core columns at the 98th and 97th floors had the total sectional area of 2,621 in^2 or 1.69 m^2. (This was summed from the released NIST SAP2000 data). There were a variety of steel grades used: 36, 42, 45 and 50 ksi yield strength. Taking 42 ksi (290 MPa) as representative, the yield load was

$$P_{y2} = 290 \text{ MPa} \times 1.69 \text{ m}^2 = 490 \text{ MN}$$

The total yield load is the sum of the above: $P_y = P_{y1} + P_{y2} = 1{,}029 + 490 = 1{,}519$ MN

The columns are stocky enough, so that a small reduction in strength, when going from yield load to buckling load can be ignored, so that we have $P_{cr} \approx 1{,}519$ MN on a static basis.

A minimum column energy absorption capacity, over a fall through one floor height, can be calculated with $\eta = 0.3$ as

$$P_{ave} = 0.3 \times 1{,}519 \text{ MN} = 455.7 \text{ MN}$$

$$\Pi_{min} = 455.7 \text{ MN} \times 3.7 \text{ m} = 1{,}686 \text{ MN-m}$$

To comport with reality, the yield load of 1,519 MN will be adjusted upwards by two factors. The first is the dynamic load factor taken as 1.1. The second is the benefit of the difference between minimum guaranteed values and the likely expected values, modestly assumed to also be 1.1. (In designing safe structures, the guaranteed minima are the right thing to use. However, for these steel types the tested yield strength is typically larger or much larger than the guaranteed values. If the objective is to find what happened, the most likely number must be used.)

To explain the above: The first factor was used to account for the dynamic strength of the steel. The load applied during impact would have been dynamic in nature and it is well understood that the elevated strain rate acts to increase the apparent yield strength. The second factor was used to account for actual yield strength as opposed to minimum values, as explained in the text. The actual values of the Twin Tower column materials were reported by [9], 1–6, Table 4–1, p. 61 to be well above the minimum guaranteed strength. Table 1 of the attached Tech Note [13] shows suggested increase factors (naming them DIF and SIF respectively) to be used for cold-formed steel design as recommended by two different ASCE publications and the DoD UFC 3-340-02.

Therefore, the total estimated buckling strength of the 97th floor columns would have been

$$P'_{cr} = 1{,}519 \text{ MN} \times 1.1 \times 1.1 = 1{,}838 \text{ MN}$$

Most columns under consideration had the SHS section for which the strength multiplier, as noted before was expected to be $\eta = 0.5$. However, most core columns had open sections, for which a smaller value would be expected. To be on the safe side, one can take $\eta = 0.4$ for all columns on the floor. Consequently, the strain energy absorption over the 3.7 m travel would be

$$\Pi = 0.4\, P'_{cr} \times 3.7 \text{ m} = 2{,}720 \text{ MN-m}$$

APPENDIX B. DIMENSIONALITY OF BUILDING COLLAPSE

There have been opinions stated by some claiming that a one-dimensional analysis such as the one presented here are not valid, because they do not include the lateral effects of the collapse, effects that were quite visible during the catastrophe. One example of such criticism is provided by Grabbe [17], who notes that there was a significant lateral ejection of dust, debris and structural fragments.

When we observe the collapsing North Tower, the downward motion seems to be quite regular in the sense that all points on the building surface move down by about the same amount in a given time period. This invites a unidirectional analytical treatment. On the other hand, the lateral effect mentioned can't be ignored, because

(a) Ejection of gas and debris causes the loss of mass and energy of the falling building.
(b) Expulsion of air, until fully accomplished, tells us that there is an air cushion in action.

Both (a) and (b) have the effect of slowing down the falling part of the building. Yet, in spite of this, the lateral action is only a minimal secondary effect. Perhaps the most rational way out of the difficulty is to say that the event is essentially one-dimensional, but the results, especially the duration of fall, need an adjustment to account for the lateral effects.

The problem is not unlike that of a steel column squashed vertically between two guided plates. In a large-deflection mode a good part of the column material will move sideways. Yet, the overall behavior can be quantified as uni-dimensional: Axial load *vs.* axial displacement.

One must observe that we know about uniform vertical movement only during the time the upper part of the building is visible and before it disappears in a cloud of dust. Later on, large structural elements can be seen falling from inside the cloud. This may suggest a loss of symmetry in the event, but then the process might have been advanced enough by that point, so that accurate preservation of symmetry may not have mattered.

APPENDIX C. THE SEQUENCE OF THE INITIAL COLLAPSE

The event may be summarized as follows. On September 11, 2001, the North Tower was impacted by a Boeing 767 airliner, which literally flew into the building. There was an immediate explosion followed by prolonged fires, which were ignited and possibly continuously fed by an estimated 38,000 liters of fuel which had been on board the aircraft (there is not unanimous agreement as to how long the fuel would have been available to sustain the fires. The NIST report and some researchers postulate that it would have burned up within minutes of the aircraft impact and thus only served to ignite the fires). After 1 hr and 42 min had elapsed, from the time of impact, the entire building collapsed, in a progressive downward movement, beginning at the 98th floor.

From an engineering viewpoint the event had many fascinating aspects. Not the least of them was "the aircraft flying into the building" mentioned above. This means the aircraft structure cut the building structure on its way. The aircraft was built of an aluminum shell on the order of 2 mm thickness, which was additionally stiffened by longitudinal and lateral elements. At contact with the building the fuselage was pitched nose down by 10.6° and hit the building between the 95th and 96th floors. There was a 25° roll, so one wing impacted a higher part of the building than the other. The exterior columns in the area of impact had 6.9 mm thick hollow square sections of 356 mm side length. Yet, the thin aluminum wings cut through the much thicker steel. The aircraft would have certainly suffered severe unspecified fractures in the process. An analysis by NIST shows the wings would have largely been disintegrated while going through the exterior wall, but that the fuselage would continue along its path with enough speed to damage some core columns on its way between the 94th to 96th floor levels. The 98th floor, where the collapse initiated, was at the edge of the impact, and was only hit by a wing tip and would have suffered no core column damage.

The Sauret video [7] quoted before is the best visual record that we have of the early phase of the fall of the North Tower as seen from the north side. It allows us, along with an NBC video [18], showing the west side, to quantify visible downward motion of the northeast, northwest, and southwest roofline corners. Measurements show them falling within 0.25 to 0.50 seconds of each other in a nearly straight downward collapse of the upper part of the building. This continues for at least two stories, after which a tilt angle of about 8 degrees southward appears while the top part of the building continues its fall. During the measurable fall, the roof is moving downward at a constant acceleration, much larger than P_{ave} of columns heated to failure would allow.

The NIST report claims that the south exterior wall failed, as a result of prolonged heating of the attached floor trusses on that side, and that this caused a propagation of failure across the 98th floor resulting in a vertical drop, but they do not analyze the spread of failure in either the horizontal or the vertical directions. They do provide the load redistribution from this south wall failure, but that does not generate loads big enough to fail the remaining columns across the floor. Bazant [2] does not explain the initial propagation of failure across the 98th floor in any of his analyses, other than to assume uniform heating and collapse. The reasons for the evenness of the initiation at the 98th floor and straight down fall in spite of asymmetric damage, the smooth motion history while falling through successive floors, and the promptness of collapse of the North Tower, remain yet to be explained.